Higher level Questions based on old exam papers

(Includes questions from 2016)

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The Rössing Foundation

Sample questions for NSSH exam paper 2 type of questions, based on the syllabus part 2

Note for the user: the question are selected from past examination papers and divided in 15 sections. Answers to these questions are also available on the website.

Questions are selected by T Hanemaaijer.

Content

Section 1 Integration

1. [Specimen paper P2 2006 Q 1]

A curve is such that $\frac{dy}{dx} = \frac{6}{x^2} + 2$. Given the curve passes through $P(3,10)$, find

(a) the equation of the curve [3]

(b) the equation of the tangent to the curve at P. [3]

2. [Paper 2 2011 Q 1]

Find $\int\left(\frac{2}{x}-\frac{4}{x^3}\right)dx$ [3]

3. [Paper 2 2011 Q3 (b)]

Find $\int\frac{2}{\sqrt{3-x}}dx$ [3]

4. [Paper 2 2011 Q 10 (b)]

A curve is such that $\frac{dy}{dx} = -x + 2$. It passes through the point (2, −1). Find the equation of the curve. [4]

5. [Paper 2 2010 Q 3 (a)]

Find

(a) $\int\frac{1}{2x-3}dx$ [2]

(b) $\int_0^2 e^{2x}\,dx$ [3]

6. [Paper 2 2013 Q 3 (b)]

(b) Find **(i)** $\int e^{2x-3}dx$ [2]

(ii) $\int_0^3\frac{1}{2x+3}\,dx$ [3]

7. [Paper 2 2014 Q 11(b)]

(b) Find $\int\frac{2}{3x-1}\,dx$. [3]

8. [Paper 2 2015 Q2]

Find $\int\left(\frac{2}{x}-\frac{4}{x^2}\right)dx$ [3]

9. [Paper 2 2016 Q 3b]

(i) Find $\int\frac{1}{2x+3}dx$ [2]

(ii) Find $\int_0^2 e^{3x-2}dx$ [2]

Section 2 Differentiation and finding gradients and determining stationary points

1. [Specimen paper P2 2006 Q 2]

Find the gradient to the curve $y = 5 \ln x - \ln 4$ at the point where $y = \ln 8$ [5]

2. [Paper 2 2011 Q 3 (a)]

Differentiate e^{x^2+1} with respect to x. [2]

3.[Paper 2 2011 Q 9]

The function f is given by $f(x) = x^2 - \dfrac{1}{x}$, $x \neq 0$.

(a) Calculate the coordinates of the stationary point. Determine the nature of the stationary point. [6]

(b) Find the coordinates of the stationary point of $y = f(x)$ and determine the nature of this point. [3]

(c) Show that the gradient of $y = f(x)$ is positive if $x > 0$. [1]

(d) Sketch and label the graph $y = f(x)$. [2]

[Remark: (a) and (b) are similar questions, a mistake from the examiner]

4. [Paper 2 2009 Q 2]

Differentiate

(a) $\dfrac{\sqrt[5]{x^3}}{3} + 10$ [2]

(b) $\dfrac{3x + 2}{x^2}$ [3]

5. [Paper 2 2009 Q 5]

The line $y = 2x + c$ is a tangent to the curve $y = 2x^2 - 6x + 20$. Find the value of c. [5]

6. [Paper 2 2010 Q 3 (b)]

Differentiate with respect to x

(a) $\ln x^3$, [1]

(b) $\ln 3x$ [1]

7. [Paper 2 2012 Q 9]

The function f is such that $f(x) = x^4 + 4x^3$

(a) Write down an expression for $f'(x)$. [1]

(b) Find the coordinates of the stationary points of $f(x)$ [3]

(c) Determine whether these stationary points are a maximum, minimum or points of inflexion. [3]

8. [Paper 2 2013 Q 3(a)]

(a) Differentiate $\ln 2x$ with respect to x. [2]

9. [Paper 2 2014 Q 2]

Differentiate

(a) $\dfrac{\sqrt[5]{x^2}}{3} + 10$ [2]

(b) $\dfrac{2x^3 - 5x}{x^2}$ [3]

10. [Paper 2 2014 Q 11(a)]

(a) Differentiate e^{x^2-3} [2]

11. [Paper 2 2015 Q 3]

(a) Differentiate $\ln(2x^2 + 3)$ with respect to x. [2]

(b) Hence find $\int \frac{x}{2x^2+3}\,dx$ [3]

12. [Paper 2 2015 Q9]

The function f is given by $f(x) = x^3 + 3x^2 + 4x + 4$.

(a) Write down an expression for $f'(x)$ and hence show that the graph of $y = f(x)$ has no stationary points. [5]

(b) Find the coordinates of the point of inflexion of the graph of $y = f(x)$. [3]

(c) Write down the coordinates of the point at which the graph of $y = f(x)$ intersect the axes. [2]

(d) Sketch and label the graph of $y = f(x)$. [2]

13. [Paper 2 2016 Q3(a)]

Differentiate e^{x^2} with respect to x. [2]

<u>Section 3 Trig equations and identities</u>

1. [Specimen paper P2 2006 Q 3]

(a) Show that the equation $2\sec^2 x + \tan x - 5 = 0$ can be written as a quadratic equation in $\tan x$. [2]

(b) Hence solve the equation $2\sec^2 x + \tan x - 5 = 0$. For (0 radians) $\leq x \leq$ (2π radians.) [5]

2. [Paper 2 2011 Q 12]

(a) (i) Prove the identity $\frac{\sin x + \cot x \cos x}{\sin x} \equiv \operatorname{cosec}^2 x$ [4]

(ii) State the value of x, in the interval $0° \leq x \leq 90°$, for which this identity is not defined. [1]

(b) (i) Show that the equation $2\tan^2\theta - 3\sec\theta = 0$ can be written as $2\sec^2\theta - 3\sec\theta - 2 = 0$ [1]

(ii) Hence, or otherwise, solve the equation $2\tan^2\theta - 3\sec\theta = 0$ in the interval $0 \leq \theta \leq 2\pi$. [4]

(c) (i) On the same diagram, sketch and label the graphs of $f(x) = -\frac{1}{2}\sin x$ and $g(x) = \cos 2x$ for $-180° \leq x \leq 0°$. [4]

(ii) Hence, state the number of solutions of the equation $f(x) = g(x)$ in the interval $0° \leq x \leq 90°$. [1]

3. [Paper 2 2009 Q 14]

(a) Prove the following identity $\frac{\sin x}{1-\cos x} - \frac{\cot x}{1} \equiv \operatorname{cosec} x$ [4]

(b) Solve the equation $\cot \frac{1}{2}\theta = -2.987$, for $0° \leq \theta \leq 360°$.
Give your answer(s) correct to the nearest degree. [3]

(c) (i) Show that the equation $5 - 7\sin x - 2\cos^2 x = 0$ can be written as $2\sin^2 x - 7\sin x + 3 = 0$. [2]

(ii) Hence, or otherwise, solve the equation $5 - 7\sin x - 2\cos^2 x = 0$ for $0 \leq x \leq 2\pi$. [3]

4. [Paper 2 2010 Q 10]

(a) Solve the equation $\sin 3x = -\frac{1}{2}$, in the interval $0 \leq x \leq \pi$. [4]

(b) Prove the identity $\tan x - \cot x \equiv \frac{1-2\cos^2 x}{\sin x \cos x}$. [3]

5. [Paper 2 2012 Q 14]

(a) Prove the identity $\sec x - \dfrac{\cos x}{1+\sin x} \equiv \tan x$ [4]

(b) Solve the equation cosec $2x = 4$ for $-\pi \leq x \leq \pi$. [3]

(c) **(i)** Show that the equation $4 - 5\cos x - 2\sin^2 x = 0$, can be written as $2\cos^2 x - 5\cos x + 2 = 0$. [2]

(ii) Hence, solve the equation $4 - 5\cos x - 2\sin^2 x = 0$ for $0^\circ \leq x \leq 360^\circ$. [3]

6. [Paper 2 2013 Q 10]

(a) Solve the equation $\cos 3x = \frac{1}{2}$ in the interval $-\pi \leq x \leq \pi$ [4]

(b) Prove the identity $\dfrac{\operatorname{cosec} x \times \tan x}{\tan x + \cot x} \equiv \sin x$ [3]

7. [Paper 2 2014 Q 14]

(a) Prove the identity $\dfrac{\tan x + 1}{\tan x - 1} \equiv \dfrac{\sec x + \operatorname{cosec} x}{\sec x - \operatorname{cosec} x}$ [4]

(b) Solve the equation $\cot 2x = 3$, for $-180^\circ \leq x \leq 180^\circ$.
Give your answer to the nearest degree. [4]

(c) Solve for x : $-2\cos^2 x - 3\sin x = 0$ for $0 \leq x \leq 2\pi$ radians [5]

8. [Paper 2 2015 Q 12]

(a) **(i)** Prove the identity $\dfrac{\cos\theta}{1+\cos\theta} + \dfrac{\cos\theta}{1-\cos\theta} \equiv 2\cot\theta \times \operatorname{cosec}\theta$ [4]

(ii) State the value of θ, in the interval $0^\circ \leq \theta \leq 90^\circ$ for which the identity is not defined. [1]

(b) **(i)** Show that the equation $\tan^2 x - \sec x = 1$ can be written as $\sec^2 x - \sec x - 2 = 0$ [2]

(ii) Hence, or otherwise, solve the equation $\tan^2 x - \sec x = 1$ for $0^\circ \leq x \leq 90^\circ$ [3]

(i) On the same diagram, sketch and label the graphs of $f(x) = -2\sin x$ and $g(x) = \cos 2x$ for the interval $0 \leq x \leq \pi$. [4]

(ii) Hence state the number of solutions of the equation $f(x) = g(x)$ in the interval $0 \leq x \leq \pi$. [1]

9. [Paper 2 2016 Q 9]

(a) Prove the identity $\dfrac{1}{\sec\theta - \tan\theta} \equiv \dfrac{1+\sin\theta}{\cos\theta}$ [3]

(b) Solve the equation $\cos 2x = -\frac{1}{2}\sqrt{3}$ in the interval $-\pi \leq x \leq \pi$. [4]

Section 4 Equations and graphs of absolute values

1. [Specimen paper P2 2006 Q 4]

(a) Sketch on the same diagram, the graph of $y = |2x-5|$ and the graph of $x + y = 4$, showing, on your diagram, the coordinates of the points of intersection of each graph with the coordinate axes. [4]

(b) Solve the equation $|2x-5| + x = 4$. [3]

2. [Paper 2 2011 Q 8]

(a) Solve the equation $|x + 3| = 2x - 3$. [4]

(b) Hence or otherwise, sketch the graph of $y = |x + 3|$ and $y = 2x - 3$ on the same system of axes. Clearly indicate all point(s) of intersection and the intercepts with the axes. [4]

3. [Paper 2 2009 Q 10]

(a) (i) Sketch the graph of $y = |-x + 3| - 1$, indicating clearly the coordinates of any points of intersection with the axes. [3]

(ii) Find the range of y. [1]

(b) Solve the inequality $|2x - 4| < 8$. [3]

4. [Paper 2 2010 Q 9]

(a) Given that $f(x) = 3 - x$ and $g(x) = |2x - 1| - |-5x|$, evaluate gf(4). [3]

(b) Sketch the graph of $y = |x - 2| + 3$. Indicate, on your diagram, the coordinates of any point(s) of intersection of the graph with the axes, as well as the coordinates of the salient point. [3]

(c) Solve the equation $|3x - 6| = x + 1$. [4]

5. [Paper 2 2012 Q 10 (a)]

Solve the inequality: $|4 - 2x| \leq 20$ [3]

6. [Paper 2 2013 Q 9]

(a) Solve the equation $9^{|1-2x|} = 3^6$ [4]

(b) Sketch the graph of $y = -|x - 2| + 1$. Indicate on your diagram, the coordinates of any point(s) of intersection of the graph with the axes, as well as the coordinates of the salient point. [4]

(c) Solve the inequality $|x - 2| < 3$ [2]

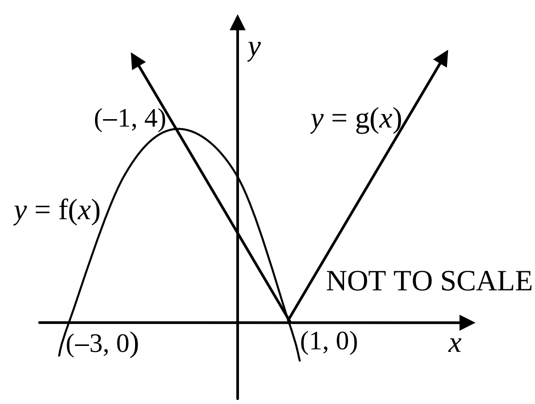
7. [Paper 2 2014 Q 10]

(a) Sketch the graph of $y = -|x + 2| + 3$, indicating clearly the coordinate of any points of intersection with the axes, as well as the salient point. [4]

(b) Solve the inequality $|3x - 2| > 4$. [3]

8. [Paper 2 2015 Q8]

Functions f and g are both defined in $\mathbb{R}$. The diagram shows the graph of the quadratic function which intersects the x-axis at $(-3, 0)$ and $(1, 0)$. The maximum point of the curve $y = f(x)$ is $(-1, 4)$. The function g is defined by $g(x) = |ax + b|$ and the graph of $y = g(x)$ passes through the points $(1, 0)$ and $(-1, 4)$.



(a) Write down the range of **(i)** f, **(ii)** g [2]

(b) Find the set of values of x for which $f(x) \leq g(x)$. [1]

(c) Find the values of a and b. [2]

(d) Determine whether or not f^{-1} and g^{-1} exists. [2]

(e) Find the equation of the function f. [4]

9. [Paper 2 2016 Q13]

(a) Solve the equation $4|2x + 3| - 8 = 36$ [3]

(b) Solve the inequality $|-3x + 2| < 1$ [3]

(c) Sketch the graph of $y = 4|2x + 3| - 8$, showing the coordinates of the points of intersection with the coordinate axes and the vertex(salient point) [4]

Section 5 Particle mechanics

1. [Specimen paper P2 2006 Q 5]

A particle moves in a straight line, so that t seconds after leaving a fixed point O, its displacement, s from O is given by $s = 10 - 10e^{-t} - \frac{1}{20}t$ Calculate:

(a) the initial velocity of the particle, [3]

(b) the value of t when the particle is instantaneously at rest, [2]
(c) the acceleration when the particle instantaneously at rest. [3]

2. [Paper 2 2009 Q 16]
An insect starts from rest on a flower F and flies in a straight line until it comes to rest on a bush B.
Its velocity, v m/s, at time t seconds after leaving F, is given by $v = 3t - t^2$.
(a) Find in terms of t, **(i)** the acceleration of the insect at time t . [1]
(ii) the displacement of the insect at time t. [2]
(b) How long does it take to reach B? [2]
(c) Find the distance between F and B. [2]
(d) Find the greatest speed of the insect between F and B. [2]

3. [Paper 2 2010 Q 4]
A particle travels in a straight line so that, t seconds after leaving a fixed point O, its velocity, v m/s, is given by $v = 35e^{-2t}$.
(a) Find an expression for the acceleration of the particle at time t. [2]
(b) Find the distance that the particle travels during the first 2 seconds. [3]

4. [Paper 2 2012 Q 16]
A particle moves in a straight line. At time t seconds its acceleration is a m/s^2, where $a = 4t - 11$.
Initially, the particle's velocity is 14 m/s and its displacement from a fixed point O on the line is 50 m.
Find
(a) the velocity of the particle at time t seconds, [3]
(b) the time when the particle is at rest, [2]
(c) the displacement of the particle at the time t seconds, [3]
(d) the distance the particle travels in the **first** second. [2]

5. [Paper 2 2007 Q 13]
A motorcyclist travels on a straight road so that, t seconds after leaving a fixed point O, his velocity, v m/s, is given by $v = 12t - t^2$.
(a) The maximum velocity is reached at a point B.
(i) Determine the maximum velocity. [3]
(ii) Calculate the distance OB. [3]
(b) On reaching B, the motorcyclist continues on the same road but slows down in such a way that, ***t* seconds after passing through *B***, its velocity v m/s is given by $v = 36 - 3t$.
He takes T seconds to travel a further 120 m after leaving B.
(i) Show that $T^2 - 24T + 80 = 0$ [4]
(ii) Hence find the value of T. [2]

6. [Paper 2 2013 Q 4]
A particle travels in a straight line so that, t seconds after leaving a fixed point O, the velocity is given by

$$v = 20e^{-3t}.$$

(a) Find an expression for the acceleration of the particle at time t. [2]
(b) Find the distance the particle travels during the first second. [3]

7. [Paper 2 2014 Q16]
A stone was thrown vertically upwards with an initial velocity of 12 m/s. It moves in such a way, that t seconds after projection, its upward velocity, v m/s is given by $\frac{dv}{dt} = -10$
(a) Show that $v = 12 - 10t$. [2]

(b) Find the height of the stone above O after t seconds. [3]

(c) (i) What was the velocity of the stone when it reached its greatest height above O? [1]

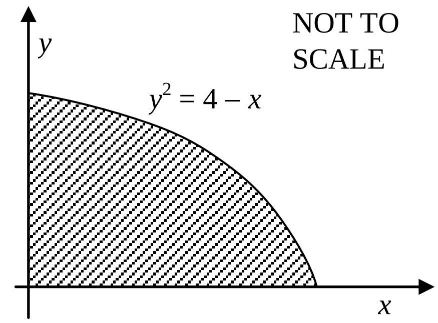
(ii) How long did the stone take to reach its greatest height above O? [2]

(iii) What was the greatest height above O? [1]

Section 6 Area and volume calculation through integration

1. [Specimen paper P2 2006 Q 6]

The diagram on the right shows part of the curve $y^2 = 4 - x$.

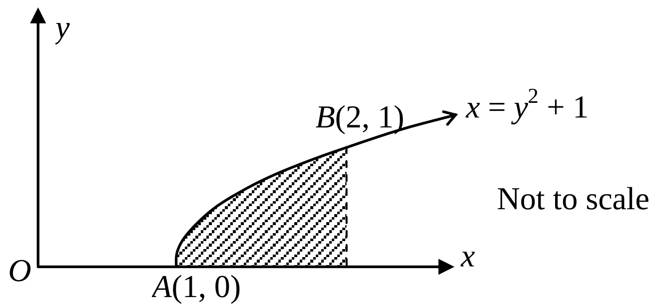


(a) Find the area of the shaded region [6]

(b) Find the volume generated when the shaded region is rotated through 360° about the x-axis. [3]

2. [Paper 2 2011 Q 10 (a)]

The diagram shows part of the curve $x = y^2 + 1$, intersecting the x-axis at $A(1, 0)$. The point $B(2, 1)$ lies on the curve.

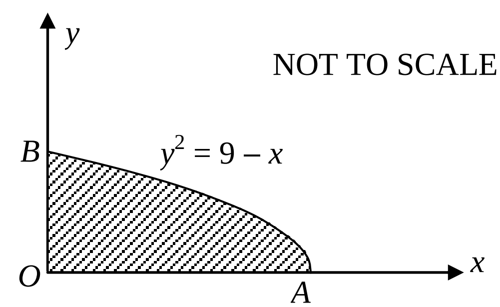


(a) The region shaded, enclosed by the curve, the x-axis and the line $x = 2$, is rotated through 360° about the x-axis.
Find, in terms of π, the volume of the solid formed. [3]

(b) Find the equation of the tangent to the curve at B. [5]

3. [Paper 2 2009 Q 8]

The graph on the right shows part of the curve $y^2 = 9 - x$, Intersecting the x-axis at A and the y-axis at B.

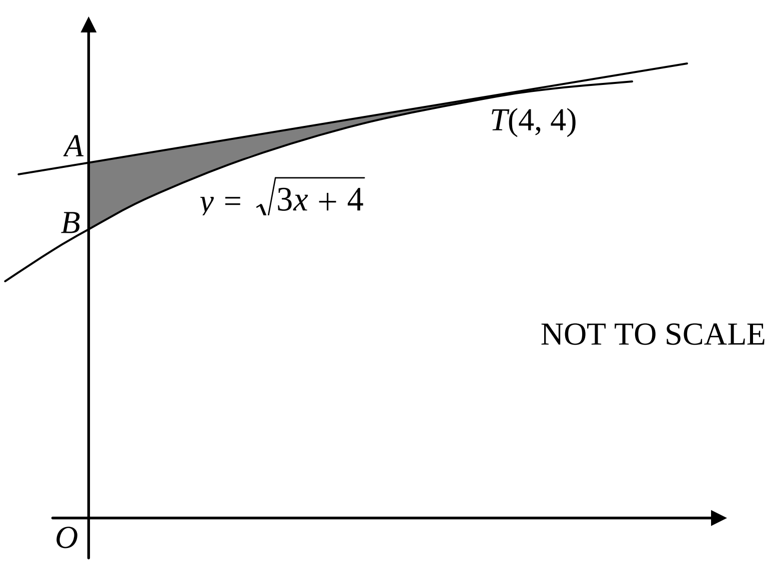


(a) Find the coordinates of A and B. [2]

(b) Find the area of the shaded region. [4]

(c) Find the volume generated when the shaded region is rotated 360° **about the *y*-axis**. Give your answer in terms of π. [4]

4. [Paper 2 2010 Q 13]



The diagrams shows part of the curve $y = \sqrt{3x+4}$, intersecting the y-axis at B.

The tangent to the curve at the point $T(4, 4)$ intersects the y-axis at A.

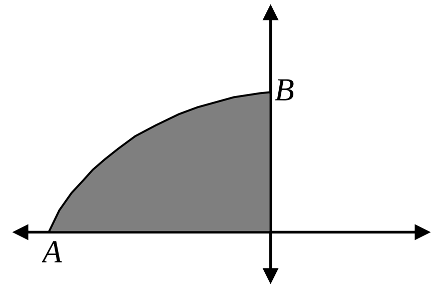
(a) Show that the equation of the tangent to the curve at the point T is
$y = \frac{3}{8}x + 2\frac{1}{2}$. [4]

(b) Find the area of the shaded region. [4]

(c) The region enclosed by the curve, the y-axis, the x-axis and the line $x = 4$ is rotated through 360° about the x-axis. Find, in terms of π, the volume of the solid formed. [3]

5. [Paper 2 2012 Q 8]

The graph shows part of the curve $y^2 = x + 16$, intersecting the x-axis at A and the y-axis at B.



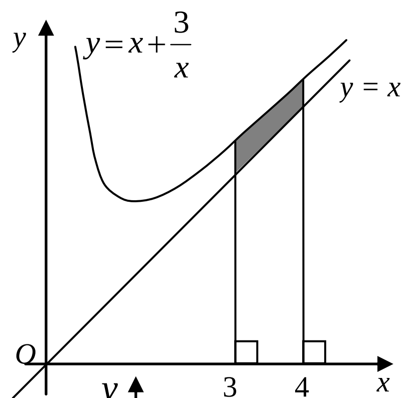
(a) Find the coordinates of A and B. [2]

(b) Find the area of the shaded region. [4]

(c) Find the volume generated when the shaded region is rotated through 360º about the y-axis. Give your answer in terms of π. [4]

6. [Paper 2 2013 Q 13]

The diagram shows part of the curve $y = x + \dfrac{3}{x}$ and the lines $y = x$, $x = 3$ and $x = 4$.



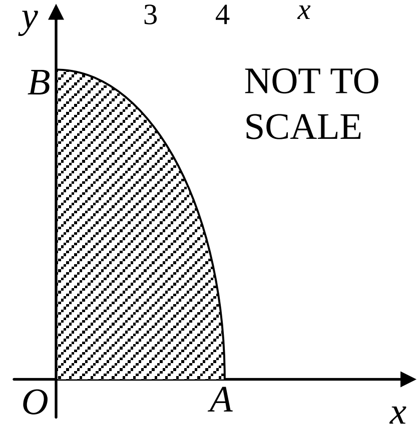
(a) Calculate the equation of the tangent to the curve where $x = 1$. [4]

(b) Find the area of the shaded region. [3]

(c) The region enclosed by the curve, the lines $x = 3$ and $x = 4$ and The x-axis is rotated through 360º about the x-axis. Find, in terms of π, the volume of the solid formed. [4]

7. [Paper 2 2014 Q 8]

The graph to the right shows part of the curve $x^2 = 16 - y$, intersecting the x-axis at A and the y-axis at B.

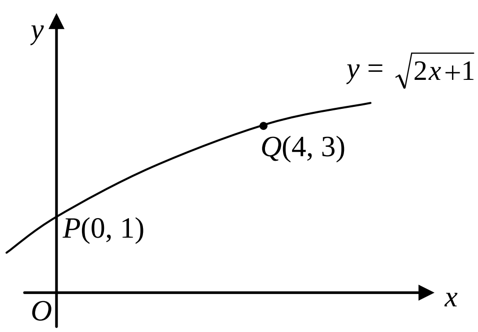


(a) Find the coordinates of A and B. [2]

(b) Find the area of the shaded region. [4]

(c) Find the volume generated, in terms of π, when the shaded region is rotated through 360º about the y-axis. [4]

8. [Paper 2 2015 Q 10]



The diagram shows part of the curve $y = \sqrt{2x+1}$, intersecting the y-axis at the point $P(0, 1)$. The point $Q(4, 3)$ lies on the curve.

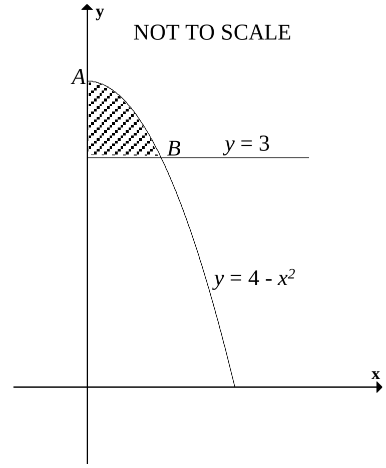
(a) Find the gradient of the tangent to the curve at Q. [3]

(b) The region enclosed by the curve, the x-axis, the lines $x = 0$ and $x = 4$ is rotated 360° about the x-axis. Find, in terms of π, the volume of the solid formed. [4]

(c) Show that the area enclosed by the line PQ and the curve from P to Q is $\frac{2}{3}$. [6]

9. [Paper 2 2016 Q12]

The diagram shows part of the curve $y = 4 - x^2$ and the line $y = 3$. The curve cuts the y-axis at A and the line $y = 3$ at B.



(a) Find the coordinates of A and B.

(b) Find the equation of the tangent to the curve at the point (2,0)

(c) Calculate the area of the shaded region.

(d) Calculate the volume obtained when the shaded area is rotated through 360° about the y-axis.

Section 7 3D vectors.

1. [Specimen paper P2 2006 Q 7]

The points A, B and C have positions vectors, relative to an origin O, of $3\mathbf{i} + 2\mathbf{k}$, $2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ and $-2\mathbf{i} + 10\mathbf{j} - 3\mathbf{k}$ respectively.

(a) Show that $\overrightarrow{BA} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. [1]

(b) Use a scalar product to find angle ABC. [6]

(c) Find the coordinates of the point D, such that $\overrightarrow{OD} = \frac{1}{5}\overrightarrow{AC}$ [2]

2. [Paper 2 2011 Q5]

The vector $\overrightarrow{AB} = -3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

(a) The point A is (2, –2, 2). Find the coordinates of the point B. [2]

(b) The vector $\overrightarrow{CD} = 9\mathbf{i} + p\mathbf{j} + 3\mathbf{k}$, where p can take different values.

(i) If AB and CD are parallel, find the value of p. [1]

(ii) If AB and CD are perpendicular, find the value of p. [2]

(iii) If $p = 11$, find the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$. [4]

3. [Paper 2 2009 Q6]

The vector $\overrightarrow{AB} = -2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ and the point A has coordinates (–3, 3, 3).

(a) Find the coordinates of point B. [1]

(b) The vector $\overrightarrow{CD} = 8\mathbf{i} - 12\mathbf{j} + t\mathbf{k}$, where t is a constant.

(i) In the case where AB and CD are parallel, find the value of t. [2]

(ii) In the case where AB and CD are perpendicular, find the value of t. [2]

(c) In the case where $t = -1$, find the angle, in degrees, between the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$. [4]

4. [Paper 2 2010 Q8]

It is given that $\overrightarrow{OA} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

(a) Calculate angle AOB. [4]

(b) Find the magnitude of $\overrightarrow{AB}$ [2]

(c) Given that $\overrightarrow{OC} = \begin{pmatrix} p \\ 3p \end{pmatrix}$, $p > 0$ and that $|\overrightarrow{OC}| = |\overrightarrow{AC}|$, calculate the value of p [3]

5. [Paper 2 2008 Q5]

The position vectors of points A, B and C, relative to an origin are $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ $\begin{pmatrix} p \\ 3 \\ p+1 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$ respectively, where p is a constant.

(a) Find the angle between position vector of A and C. [3]

(b) Obtain an expression, in terms of p, for $\overrightarrow{AB} \cdot \overrightarrow{AC}$ [4]

(c) Find the value of p for which angle $BAC = 90^\circ$. [2]

6. [Paper 2 2012 Q6]

The points A, B and C have position vectors $-\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $2\mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} - \mathbf{j}$ respectively.

(a) Write down the coordinates of point C. [1]

(b) Find expressions for the vector $\overrightarrow{BA}$ and the vector $\overrightarrow{BC}$ [4]

(c) Calculate the angle ABC, in radians. [4]

7. [Paper 2 2013 Q8]

The points A, B, C and D have position vectors $\begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ c \\ -1 \end{pmatrix}$ and $\begin{pmatrix} d \\ -2 \\ 1 \end{pmatrix}$ respectively, relative to an origin O.

(a) Calculate angle AOB correct to the nearest degree. [4]

(b) Find the value of c so that angle BAC is 90°. [4]

(c) Find the value of d such that $|\overrightarrow{BD}| = 2$ [2]

8. [Paper 2 2014 Q6]

The vector $\overrightarrow{OA}$ is given by $\overrightarrow{OA} = 4\mathbf{i} - 2\mathbf{j} + 2\sqrt{11}\mathbf{k}$, where O is the origin.

(a) Find the length of OA. [2]

(b) The line OA is produced to B, where OB = 16 units. Find the coordinates of the point B. [2]

(c) The vector $\overrightarrow{OC} = 2\mathbf{i} + 2p\mathbf{j}$, where p is a constant. Using a scalar product to OC,

(i) find the value of p for which angle AOC is a right angle, [2]

(ii) find the two values of p for which the angle $AOC = 60^\circ$. [4]

9. [Paper 2 2015 Q5]

Relative to an origin O, the points A, B and C have position vectors $\begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ c \\ 1 \end{pmatrix}$ respectively.

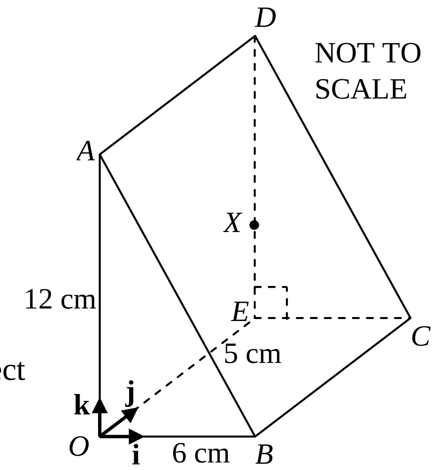
(a) Calculate angle AOB, correct to the nearest degree. [4]

(b) Find the value of c for which the angle $BAC = 90^\circ$. [5]

10. [Paper 2 2016 Q8]

The diagram shows a right-angled triangular prism, in which $OB = 6$ cm, $OA = 12$ cm and $OE = 5$ cm. X is a point on ED such that $EX = 3$ cm.

Unit vectors **i**, **j** and **k** are parallel to OB, OE and OA respectively.



(a) Find **(i)** vector OX, [1]

(ii) vector XB, [2]

(iii) vector AC. [2]

(b) By using a vector method, calculate angle OXB in radians, correct to 2 decimal places.

Section 8 Factor and remainder theorem

1. [Specimen paper P2 2006 Q 8]

(a) Show that $(x + 3)$ is a factor of $x^3 + 2x^2 - 7x - 12$ and hence solve the equation $x^3 + 2x^2 - 7x - 12 = 0$ giving your answer correct to two decimals places where necessary. [5]

(b) The polynomial $f(x) = x^3 + ax^2 + bx - 3$ has a factor $(x + 1)$. When $f(x)$ is divided by $(x + 2)$, it has the same remainder as when it is divided by $(x - 2)$. Find the value of a and b. [4]

2. [Paper 2 2011 Q2]

A function f is such that $f(x) = 6x^3 - 13x^2 - 10x + 24$.

(a) Given that $f(x) = (x + 1)(ax^2 + bx + c) + R$, find the value of a, b, c and R. [4]

(b) Show that $(x - 2)$ is a factor of $f(x)$. [2]

(c) Hence, factorise $f(x)$ completely. [3]

3. [Paper 2 2009 Q1]

It is given that $f(x) = 2x^3 + 3x^2 - 5x - 6$

(a) Show that $(x + 1)$ is a factor of $f(x)$. [2]

(b) The *polynomial* can also be written as $(x + 1)(ax^2 + bx + c)$. Find the values of a, b and c. [3]

(c) Hence or otherwise, solve the equation $f(x) = 0$. [3]

4. A quadratic expression is given by $x^2 - px + 1$, where p is a constant.

(a) When $x^2 - px + 1$ is divided by $(x + 2)$ the remainder is 3. Find the value of p. [3]

(b) **(i)** In the case where $p = 3$, solve the equation $x^2 - px + 1 = 11$. [2]

(ii) Hence, or otherwise, find the value of a which solves the equation $\left(\sqrt{a-1}\right)^2 - 3\sqrt{a-1} = 10$ [2]

5. [Paper 2 2010 Q2]

The function f is given by $f(x) = x^3 + 2x^2 + ax - 8$ where a is a constant. When $f(x)$ is divided by $(x - 2)$ the remainder is -6.

(a) Show that $a = -7$ [2]

(b) **(i)** Show that $(x + 1)$ is factor of $f(x) = x^3 + 2x^2 + ax - 8$. [1]

(iii) It is also given that $f(x)$ can be written in the form $(x + 1)(x^2 + bx + c)$ where b and c are constants. Find the value of b and c. [2]

(iii) Solve the equation $f(x) = 0$, giving your answer correct to 2 decimal places where necessary. [3]

(c) **(i)** Find the x-coordinates of the turning points of the graph of $y = f(x)$. [2]

(ii) Determine the nature of each of the turning points. [3]

6. [Paper 2 2010 Q7]

(a) The function f is defined by f : $x \mapsto \ln(x - 2)$, for the domain $x > a$, where a is a constant.

(i) Find the minimum value of a. [1]

For this value of a find,

(ii) The range of f, [1]

(iii) an expression for f^{-1}, [2]

(iv) sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram. Labeling any points of intersection Of the curves with the axes. [4]

(b) The function g is defined by g : $x \mapsto x + 3$, $x \in \mathbb{R}$.

(i) Evaluate gf(4), giving your answer correct to two decimal places. [2]

(ii) Show that fg(–3) cannot be evaluated. [1]

7. [Paper 2 2013 Q2]

The function f is given by $f(x) = -x^3 + x^2 + 5x + c$, where c is a constant.

When $f(x)$ is divided by $(x + 2)$, the remainder is 5.

(a) Show that $c = 3$. [2]

(b) **(i)** Show that $(x + 1)$ is a factor of $f(x) = -x^3 + x^2 + 5x + 3$. [2]

(ii) Factorise $f(x)$ fully. [4]

(c) **(i)** Find the x-coordinates turning point of the graph $y = f(x)$ [2]

(ii) Determine the nature of each of the turning points. [3]

8. [Paper 2 2014 Q1]

It is given that $f(x) = 10x^3 - x^2 - 17x - 6$.

(a) Show that $(x + 1)$ is a factor of $f(x)$ [2]

(b) Given that $f(x)$ can also be written as $(x + 1)(ax^2 + bx + c)$. Find the values of a, b and c. [3]

(c) Hence, or otherwise, solve the equation $f(x) = 0$. [3]

9. [Paper 2 2014 Q4]

It is given that $f(x) = x^2 - px - 1$, where p is a constant.

(a) When $x^2 - px - 1$ is divided by $(x - 3)$, the remainder is –12. Find the value of p. [3]

(b) **(i)** In the case where $p = 1$, solve the equation $x^2 - px - 1 = 5$ [2]

(ii) Hence, or otherwise, solve the equation $\left(\sqrt{a-2}\right)^2 - \sqrt{a-2} - 1 = 5$. [2]

10. [Paper 2 2015 Q1]

A function f is such that $f(x) = 6x^3 - 7x^2 - 28x + 20$

(a) Given that $f(x) = (ax^2 + bx + c)(x + 1) + R$, find the values of a, b, c and R. [4]

(b) Show that $(x + 2)$ is a factor of $f(x)$. [2]

(c) Hence factorize $f(x)$ completely. [3]

11. [Paper 2 2016 Q2]

The function f is given by $f(x) = x^3 - 7x + c$, where c is a constant. When $f(x)$ is divided by $(x - 2)$, the remainder is −12.

(a) Show that $c = -6$. [2]

(b) Factorise $f(x)$ fully. [4]

(c) **(i)** Find the x-coordinates of the turning points of the graph of $y = f(x)$. [2]

(ii) Determine the nation of each of the turning points. [3]

Section 9 Trigonometric functions

1. [Specimen paper P2 2006 Q 9]

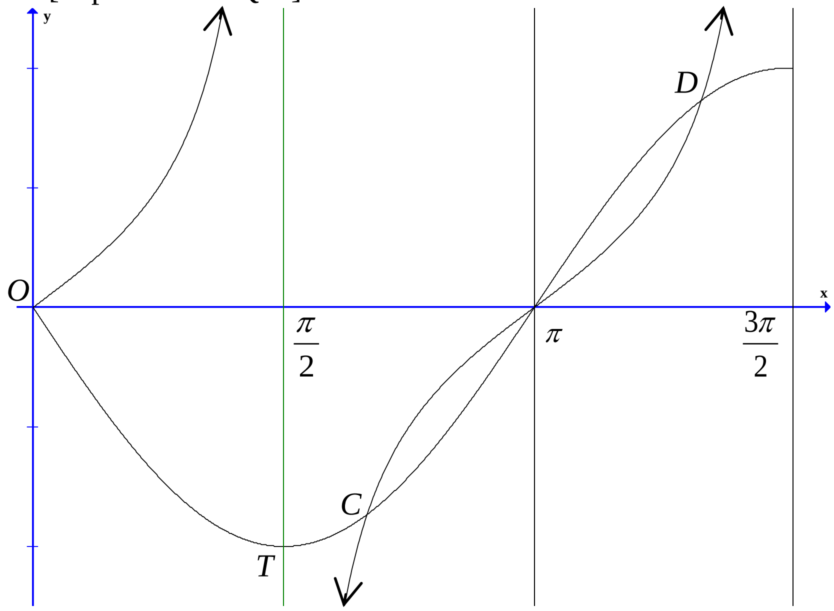
The function f is defined by f: $x \mapsto 5 - 4\cos 2x$, for $0^\circ \le x \le 360^\circ$.

(a) State the amplitude and period of f. [2]

(b) Sketch the graph of $y = \text{f}(x)$ and write down the coordinates of the maximum points. [5]

(c) Find the smallest value of x that satisfies the equation $\text{f}(x) = 3$. [3]

2. [Paper 2 2009 Q13]



Functions f and g are defined by

$$\text{f}: x \mapsto -2\sin x,\ 0 \le x \le \frac{3\pi}{2},$$

$$\text{g}: x \mapsto \tan x,\ 0 \le x \le \frac{3\pi}{2}.$$

The diagram to the right shows the curves $y = \text{f}(x)$ and $y = \text{g}(x)$ intersecting at O, C and D.

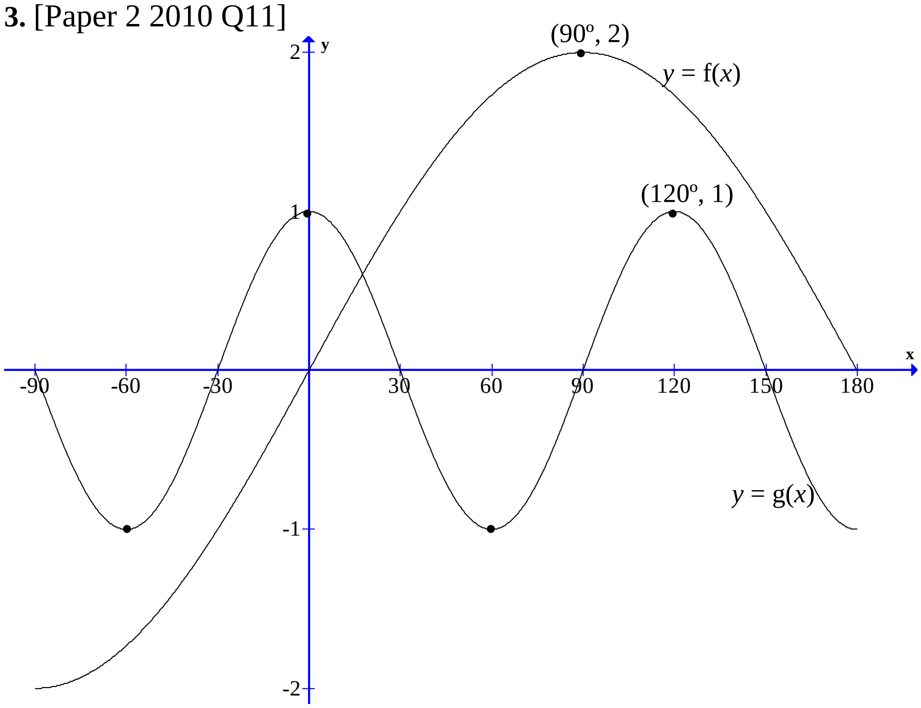
(a) Find the coordinates of T, the turning point of the curve $y = \text{f}(x)$. [1]

(b) Write down the period of g. [1]

(c) Write down the range of f. [1]

(d) If C has coordinates $\left(\frac{2}{3}\pi, -\sqrt{3}\right)$, write down the coordinates of D. [2]

3. [Paper 2 2010 Q11]



(a) The function f is given by $\text{f}(x) = a\sin bx$, where a and b are constants.

(i) Find the value of a and b for $-90 \le x \le 180^\circ$ [2]

(ii) Find the range of the function f for $-90 \le x \le 180^\circ$. [1]

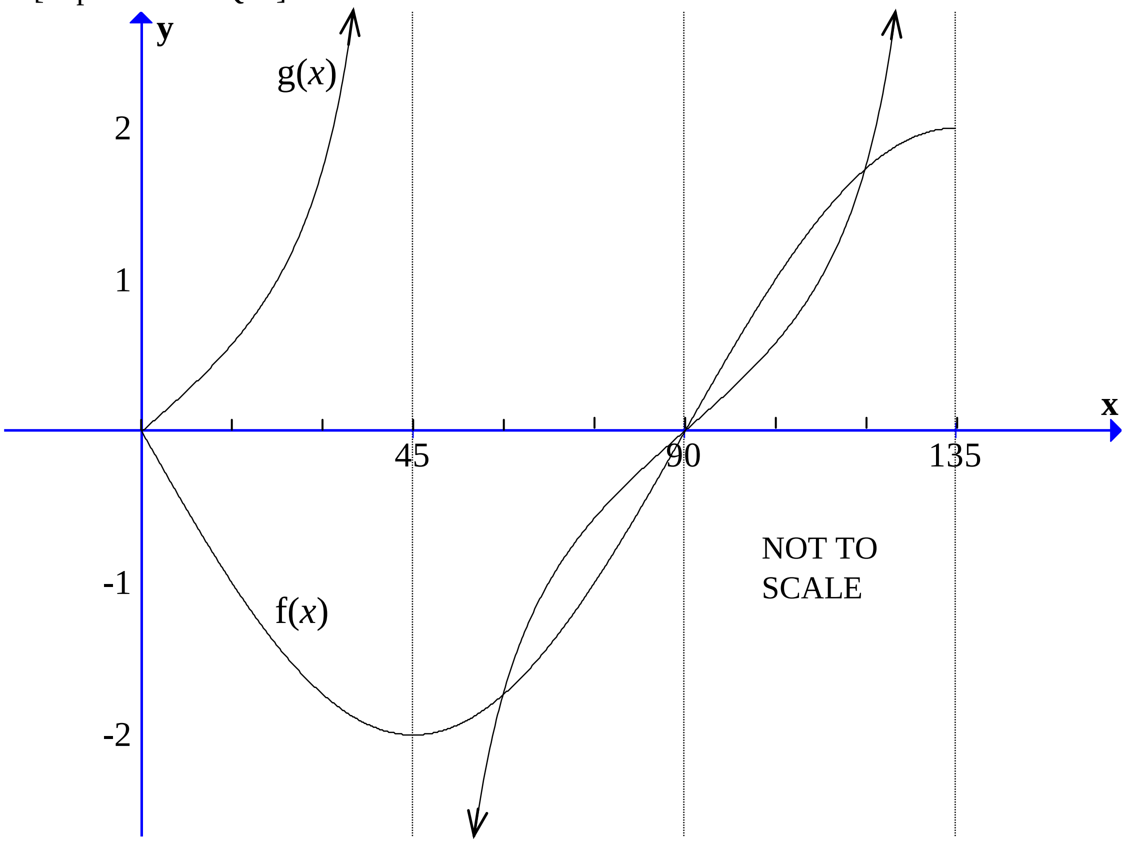
(b) The function g is given by $\text{g}(x) = p\cos qx$, where p and q are constants.

(i) Find the value of p and q in the given interval $-90 \le x \le 180^\circ$ [2]

(ii) Find the period of the function g in the given interval $-90 \le x \le 180^\circ$. [1]

(c) Use the diagram to find the range of x-values for which $\text{f}(x)\text{g}(x) < 0$ in the given interval $-90 \le x \le 180^\circ$. [3]

4. [Paper 2 2012 Q13]

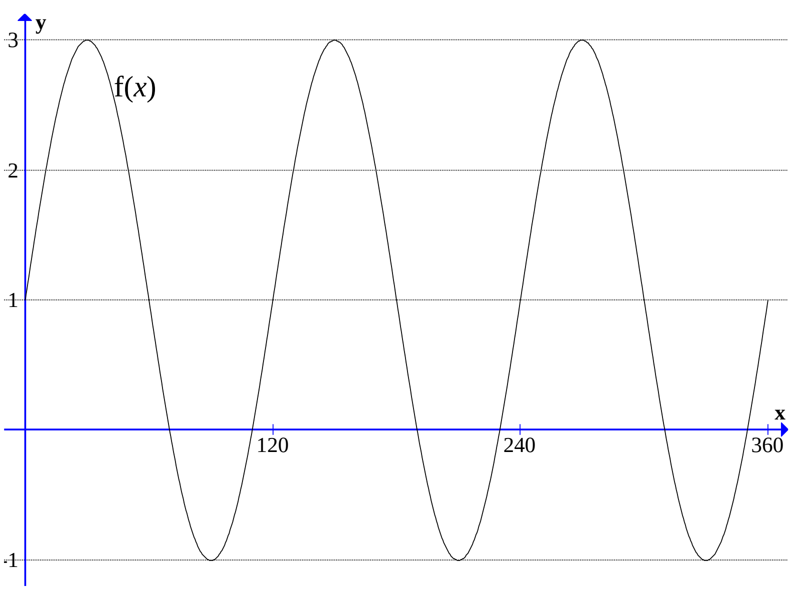


Functions f and g are defined by $f : x \mapsto a \sin bx$, for $0^\circ \le x \le 135^\circ$ and
$g : x \mapsto \tan cx$ for $0^\circ \le x \le 135^\circ$.

(a) Find the values for *a*, *b* and *c*. [3]

(b) Write down the equation of the reflection of the graph, g(*x*), in the *x*-axis. [1]

5. [Paper 2 2007 Q11]



(a) The diagram shows part of a trigonometric function defined by
$f : x \mapsto a \sin (bx) + c$, for $0^\circ \le x \le 360^\circ$.

(i) Find the values of the positive integers *a*, *b* and *c*. [3]

(ii) Find the smallest value of *x* for which f(*x*) = 0. [2]

(b) Given that $p = 4 \sin x - 3 \cos x$ and $q = 4 \cos x + 3 \sin x$,

(i) find the value of the acute angle *x*, in radians, for which $p = q$, [3]

(ii) show that $p^2 + q^2$ is constant for all values of *x*. [3]

6. [Paper 2 2013 Q 11]

The diagram shows the graphs of $y = a \sin bx$ and $y = c \cos dx$ for $0° \leq x \leq 180°$. $(0, -2)$ and A are stationary points.

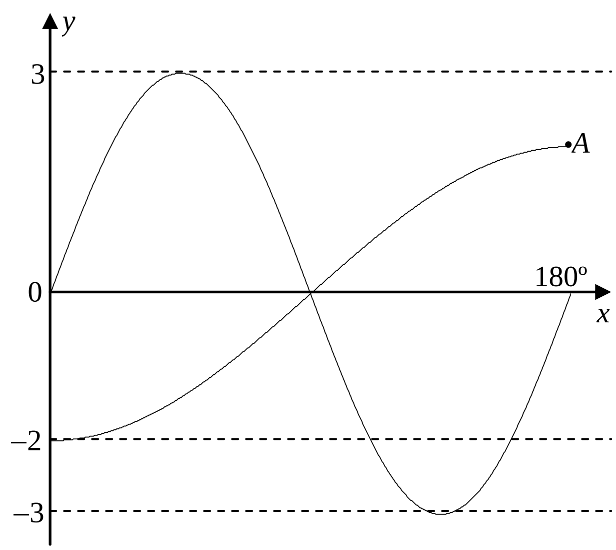
(a) Determine the values of a, b, c and d. [4]

(b) Find the coordinates of the point A on the diagram. [1]

(c) Find the period of $y = a \sin bx$. [1]

(d) Determine the range of $y = c \cos dx$ [1]

(e) Write down the new equation of $y = a \sin bx$, when the y-axis shifted 45° to the right. [1]



7. [Paper 2 2014 Q 13]

(a) The functions f and g are such that $f(x) = \sin x + 1$, for $-90° \leq x \leq 270°$

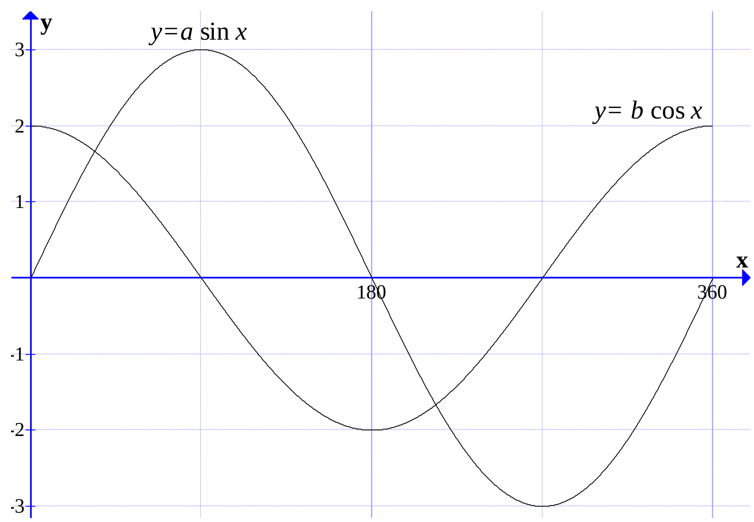
$g(x) = \cos 2x$, for $-90° \leq x \leq 270°$

Draw sketch graphs of $y = f(x)$ and $y = g(x)$ on the same system of axes. [4]

(b) Find the range of values where $f(x) \times g(x) < 0$ for $-90° \leq x \leq 0°$ [1]

8. [Paper 2 2016 Q 10]

The diagram shows the graphs of $y = a \sin x$ and $y = b \cos x$ for $0° \leq x \leq 360°$.



(a) Determine the values of a and b. [2]

(b) Solve the equation $a \sin x = b \cos x$ for $0° \leq x \leq 360°$. [3]

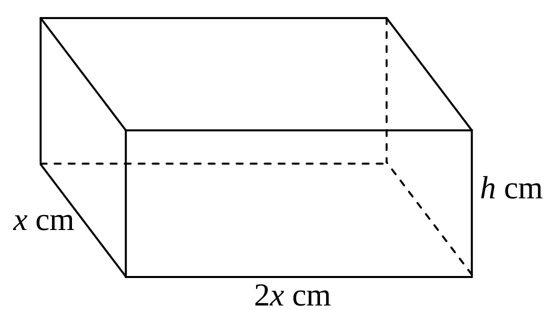
(c) Write down the range of values of x for which $a \sin x < b \cos x$ for $0° \leq x \leq 360°$. [2]

(d) The y-axis is moved (or shifted) 180° to the right but the curves remain in their original position.
Write down the equation of the curve that originally had the equation of $y = b \cos x$. [1]

<u>Section 10 Finding optimal solutions and rate of change</u>

1. [Specimen paper P2 2006 Q 10]

A solid rectangular block has a base, which measures $2x$ cm by x cm. The height of the block is h cm and the volume is 72 cm^3.



(a) Express h in terms of x. [2]

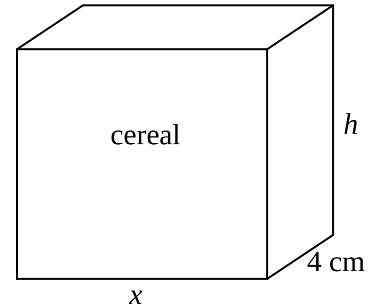
(b) Show that the total surface area, A cm^2, of the block in terms of x, is $A = 4x^2 + \dfrac{216}{x}$ [2]

(c) Given that x can vary, calculate the value of x for which A has a stationary value. [3]

(d) Find this value of A and determine whether it is a maximum or a minimum. [3]

2. [Paper 2 2011 Q4]

A cereal box has the shape of a rectangular prism as shown in the diagram. The box has a volume of 480 cm^3, a breadth of 4 cm, a length of x cm and a height of h cm.



(a) Express the height h in terms of x. [2]

(b) Show that the total surface area of a box, A cm^2, is given by

$A = 8x + \dfrac{960}{x} + 240$ [2]

(c) The company which uses these boxes requires the dimensions to be such that the minimum amount of carton is used in the production. Given that x can vary, determine the value of the minimum area of cardboard used, correct to the nearest integer.

3. [Paper 2 2009 Q3]

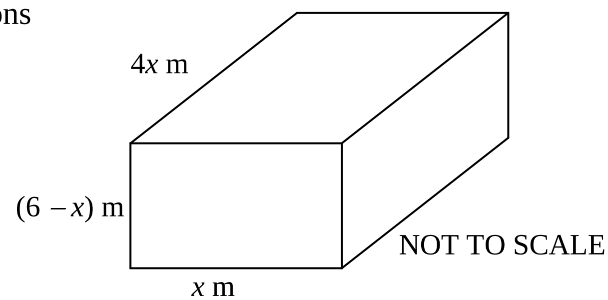
A water tank has an inlet pipe as well as an outlet pipe. Both of these regulate the depth of the water in the tank. The depth of the water is given by the function $h(t) = 3 + \frac{1}{2}t^2 - \frac{1}{4}t^3$, where h($t$) is the depth of water in meters and t is the times in hours after 08:00.

(a) Determine the rate at which the depth of the water in the tank is changing at 11:00. State whether the depth is increasing or decreasing at this time. [3]

(b) At what time will the rate of water flowing into the tank, be the same as the rate of water flowing out of the tank? [3]

4. [Paper 2 2010 Q5]

The diagram shows a rectangular box of which the dimensions Are $4x$ meters, x meters and $(6 - x)$ meters respectively.



(a) Find, in terms of x, an expression for the volume V m^3, of the box. [2]

(b) **(i)** Given that x can vary, calculate the value of x for which the volume V is a maximum. [3]

(ii) Hence, find the maximum volume of the box. [1]

5. [Paper 2 2012 Q3]

During an experiment the temperature, T (in degrees Celsius), varies with time, t (in hours), according to the formula $T = 15 + 2t - \dfrac{1}{4}t^2$ for $0 \le t \le 10$.

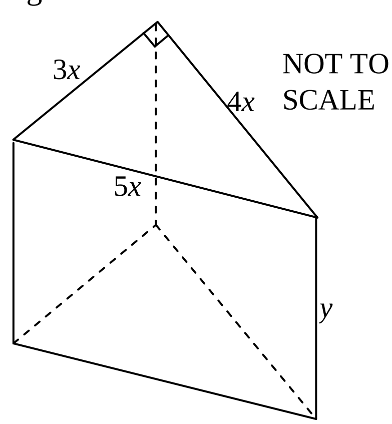
(a) Determine an expression for the rate of change of the temperature with respect to time. [2]

(b) **(i)** Find the interval (in hours) during which the temperature T was dropping. [2]

(ii) Find the corresponding range of temperature in degree Celsius. [2]

6. [Paper 2 2013 Q5]

The diagram shows a right-angled triangular wooden block.
The ends are right angled triangles with sides $(3x)$ cm, $(4x)$ cm and $(5x)$ cm respectively.
The other three faces are rectangular.
The height of the block is y; the total surface area of the block is 3600 cm^2.



(a) Show that $y = \dfrac{300 - x^2}{x}$ [3]

(b) Find the value of x for which the block will have a maximum **volume.** [4]

7. [Paper 2 2014 Q3]

Andrew raised his hand during a calculus lesson on sketching of cubic graphs. He said that he had found a general formula for the *x*-coordinate of the turning points of the graph $y = ax^3 + bx^2 + cx + d$.

His formula was as follows: $x = \dfrac{-b \pm \sqrt{b^2 - 3ac}}{3a}$

It is interesting to see that this is very similar to the quadratic formula which we know very well.

(a) Find $\dfrac{dx}{dy}$ and thus derive Andrew's formula. [3]

(b) Using this formula, or otherwise, prove that the graph $y = x^3 + 2x^2 + 10x - 18$ has no turning points. [3]

8. [Paper 2 2014 Q9]

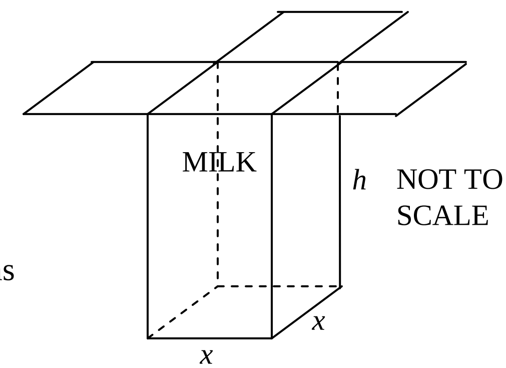
A biologist deduces that when a bactericide is introduced into a culture of bacteria, the number of bacteria present is given by $B(t) = 1000 + 50t - 5t^2$, where $B(t)$ is the number of bacteria , in millions, present *t* hours after the bactericide was introduced into the culture.

(a) Calculate the rate of change of the number of bacteria when $t = 3$. [2]

(b) When does the population of bacteria start decreasing? [2]

(c) When will the whole culture of bacteria be exterminated? [2]

9. [Paper 2 2015 Q4]



A rectangular container has a square base and, when full, contains 1 litre of milk. Three of the vertical faces have flaps, each of which has the same area as the base. The length of the side of the square is *x* cm and the height of the container is *h* cm. [1 litre = 1000 cm^3]

(a) Find an expression for the area, *A* cm^2, of cardboard used in terms of *x* and *h*. [2]

(b) Show that $A = 4x^2 + \dfrac{4000}{x}$ [2]

The company which produces these containers requires the dimensions to be such that the minimum amount of cardboard is used in the production. [2]

(c) Given that *x* may vary, determine the value of *x* and *h* for which the area of the cardboard used in the Production of the container is a minimum. [5]

10. [Paper 2 2016 Q4]

In order to reduce the temperature in a room from 30°C, a cooling system is allowed to operate for 10 minutes.

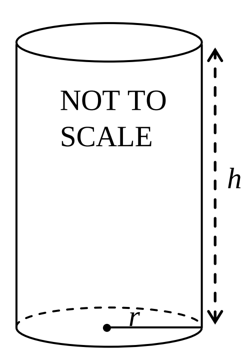
The room temperature, *T*°*C* after *t* minutes is given in degrees Celsius by the formula:

$T = 30 - 0.008t^3 - 0.16t$ for $0 \leq t \leq 10$.

(a) At what rate is the temperature changing when $t = 5$? [3]

(b) Calculate the room temperature at $t = 10$. [2]

11. [Paper 2 2016 Q5]



The diagram shows a cylindrical cool drink tin, which has a capacity of 350 ml.

(a) Express the height, *h* cm in terms of the radius *r* cm of the cool drink tin. [1]

(b) Show that the total surface area, *A* cm^2, of the cool drink in terms of *r* is given by $A = 2\pi r^2 + \dfrac{700}{r}$ [2]

(c) Given that *r* can vary, determine the radius for which the total surface area of the can will be a minimum. [4]

<u>Section 11 Solving logarithmic and exponential equations; logarithmic inequalities and log graphs.</u>

1. [Specimen paper P2 2006 Q 11 (a)]

Solve the equation $\log_2 x - 3\log_x 2 = 2$.

2. [Paper 2 2010 Q 6]

(a) Solve the equation $2\log x - \log_x 10 = 1$ [5]

(b) The graph of log *y* against *x* is a straight line with gradient 2 and it passes through the point (0, 3).
The variables *x* and *y* are related by the equation $y = ab^x$, where *a* and *b* are constants.
Find the values of *a* and *b*. [5]

3. [Paper 2 2012 Q10 (b)]

Sketch the graph of $y = |\log_3 x|$. Write the letters *A* and *B* on the graph, to show where one would read off the solutions to $|\log_3 x| = 1$. [3]

4. [Paper 2 2012 ?]

The variables *P* and *x* are related by the equation $P = \log_3(x-2)+\log_3(2x+1)$.

(a) Determine the values of *x* for which *P* is defined. [2]

(b) Solve for *x* if *P* = 1. [4]

5. [Paper 2 2008 Q 6 (a)]

Solve the inequality $\log(12x + 4) > 1 + \log(x + 2)$. [4]

6. [Paper 2 2013 Q 6]

(a) Solve the equation $3\log p + \log_p 100 = 5$. Show all your working. [5]

(b) The graph of log *y* against log *x* is a straight line with a gradient –½ and it passes through (2, 5).
The variables *x* and *y* are related by the equation $y = ax^b$, where *a* and *b* are constants.
Find the values of *a* and *b*. [5]

7. [Paper 2 2014 Q 15]

Given: $3\log_8 x - 5 = 2\log_x 8$.

(a) Write down the values of *x* for which $\log_x 8$ is defined. [1]

(b) Solve the equation. [6]

8. [Paper 2 2015 Q 7]

(a) Solve the inequality $\log_{\left(\frac{1}{2}\right)} x + \log_{\left(\frac{1}{2}\right)}(x+1) \geq -1$ [4]

(b) (i) Solve the equation $y^2 - 9y - 2 = 0$, correct to 2 decimal places, show all your working. [3]

(ii) Hence solve the equation $3^{2x} - 3^{x+2} - 2 = 0$. [3]

9. [Paper 2 2016 Q6]

(a) Solve the equation $3^{-2x} - 3^{2-x} + 18 = 0$ by using the substitution $k = 3^{-x}$, or otherwise. [5]

(b) Solve the equation $\log_3(x+1)+\log_{\frac{1}{3}}(2x-1)=1$ [4]

Section 12 Relation between variables

1. [Specimen paper P2 2006 Q 11 (b)]

The variables x and y are related by the equation $y = ae^{bx}$, where a and b are constants.

(a) Show that the graph of ln y against x is a straight line and express the gradient and the intercept on the ln y-axis in terms of a and/or b. [3]

(b) Given that this line passes through (0, 0.6) and (2, 1.6) find the value of a and b. [4]

(c) Find the value of y when $x = 4$. [1]

2. [Paper 2 2011 Q 6]

The variables x and y are related by the equation $y = 3e^{5x}$.

(a) Express this equation in the form $\ln y = mx + c$, stating the values of m and c. [2]

(b) **(i)** Sketch the graph of ln y against x. [3]

(ii) Write down the coordinates of the point of intersection with the ln y-axis. [1]

(c) Find the value of **(i)** y when $x = -1$. [1]

(ii) x when $y = 90$. [3]

3. [Paper 2 2009 Q 15]

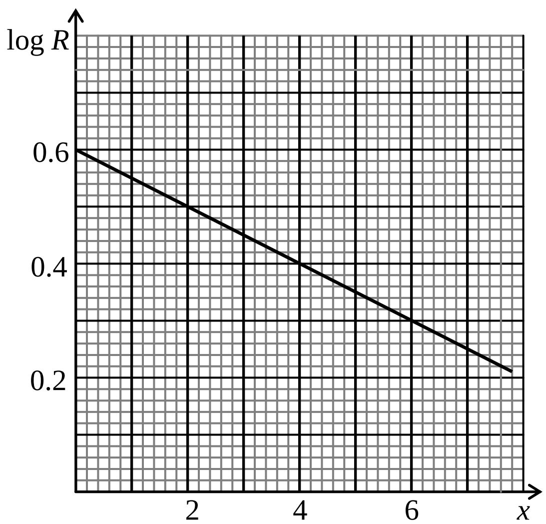
Given that the variables y and x are related by the equation $y = \log_2(x+3) + \log_2(x-3)$,

(a) Determine the values of x for which y is defined, [2]

(b) Find the value of x when $y = 4$, [3]

(c) Solve the inequality $y > 4$. [2]

4.



The variables R and x are related by $R = ka^x$, where k and a are constants.

The graph shows the linear relationship between log R and x.

The line meets the log R axis at 0.6 and log R = 0.4 when $x = 4$.

(a) Calculate the values of k and a in the formula $R = ka^x$, correct to 2 significant figures. [5]

(b) Use your values of k and a to calculate the value of R when $x = 10$. [1]

Section 13 Functions, inverse and composite functions

1. [Specimen paper P2 2006 Q 12]

The function f is defined by $\text{f} : \mapsto 2x^2 + 8x - 10$

(a) Express $2x^2 + 8x - 10$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(b) Hence, or otherwise, state the least value of y and the corresponding value of x. [2]

(c) Find the value of the constant k for which the equation $2x^2 + 8x - 10 = k$ has real, distinct roots. [3]

The function g is defined by $\text{g} : x \mapsto 2x^2 + 8x - 10$ for $x \geq -2$

(d) Explain why g has an inverse but f has not. [1]

(e) Find an expression for g^{-1}. [3]

(f) State the domain and range of g^{-1}. [2]

2. [Paper 2 2011 Q11 (b)]

The functions f and g are defined by f : $x \mapsto \dfrac{1-2x}{3+x}$, for $x \neq -3$ and g : $x \mapsto \ln(x-3)$ for $x > 3$.

(a) Evaluate fg(5), giving your answer correct to 3 decimal places. [2]

(b) Find an expression for ff(x), expressing your answer in the form $\dfrac{a+bx}{c+dx}$, where a, b, c and d are integers [3]

(c) Explain why gf(0) cannot be evaluated. [3]

(d) Find an expression for $f^{-1}(x)$ and $g^{-1}(x)$. [5]

3. [Paper 2 2009 Q9]

The function f is defined by $f(x) = -\log_2 x$, for $x > 0$.

(a) Sketch the graph of $y = f(x)$, labeling any intercepts with the coordinates axes. [2]

(b) The function h is a reflection of f in the x-axis.

(i) Sketch the graph of $y = h(x)$ on the same diagram that was drawn for **part (a)**. [1]

(ii) Express h in terms of x. [1]

(c) Find an expression for $f^{-1}(x)$, in terms of x. [2]

4. [Paper 2 2009 Q11]

The functions f and g are defined by f: $x \mapsto e^{x+1}$, $x \geq 0$. g: $x \mapsto 3 - x$, $x \in \mathbb{R}$

(a) Write down the range of f. [1]

(b) Find an expression for f^{-1}, in terms of x. [2]

(c) Evaluate fg(–4). [2]

5. [Paper 2 2012 Q7]

It is given that f : $x \mapsto 4x - 1$, $x \in \mathbb{R}$ and g : $x \mapsto x^2$ $x \in \mathbb{R}$

(a) Find the value(s) of a such that fg(a) = 99 [3]

(b) Find the value(s) of b such that $gf^{-1}(b) = 81$ [4]

(c) Find the value(s) of c such that ff(c) = 27 [2]

(d) State with a reason whether g has an inverse. [1]

6. [Paper 2 2013 Q7]

The function f is such that $f(x) = \log_2(x-3)$, for the domain $x > a$, where a is a constant.

The function g is such that $g(x) = 2x - 3$, $x \in \mathbb{R}$.

(a) Find the minimum value of a. [1]

(b) For the value of a in **(a)**,

(i) find the range of f. [1]

(ii) find an expression for f^{-1} [2]

(c) On the same diagram, sketch and label, the graphs of $y = f(x)$ and $y = f^{-1}(x)$. Label any points of Intersection of the curves with the x-axes. [4]

(d) Evaluate gf(7). [2]

(e) Show that fg(1) cannot be evaluated. [1]

7. [Paper 2 2015 Q 11]

Functions f and g are defined by f: $x \mapsto \dfrac{x-2}{x+3}$, for $x \neq -3$ and g: $x \mapsto \ln(x+1)$ for $x > -1$.

(a) Evaluate fg(2), giving your answer correct to 1 decimal place. [2]

(b) Explain why gf(–1) cannot be calculated. [2]

(c) Find an expression for **(i)** f^{-1} [3]

(ii) g^{-1} [2]

(d) Sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$ on the same set of axes, labeling each graph clearly. [4]

8. [Paper 2 2016 Q7]

(a) The function f is defined by f: $x \mapsto e^x + 3$, $x \in \mathbb{R}$.

(i) Find the range of f. [1]

(ii) Find an expression for f^{-1}. [2]

(iii) Find the domain for f^{-1}. [1]

(iv) Sketch and label the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same system of axes, labeling any points of intersection of the curves with the axes. [4]

(b) The function g is defined by g: $x \mapsto 3x - 2$, $x \in \mathbb{R}$.

(i) Evaluate gf(4), giving your answer correct to 1 decimal place. [2]

(ii) Show that $f^{-1}g(-1)$ cannot be evaluated. [2]

Section 14 Sequences & patterns and series

1. [Specimen paper P2 2006 Q 13(a)]

Evaluate $\displaystyle\sum_{r=0}^{23}(45 - 3r)$ [5]

2. [Specimen paper P2 2006 Q 13(b)]

Copper is extracted from its ore obtained from a mine in Namibia. During the first year of operation, the ore mined yielded 7 000 kg of copper. With the increased difficulty of mining, the production of copper in each subsequent year shows a decrease of 20% on the previous year's production. Assuming that mining continues in the same way for an infinite period of time,

(a) calculate the maximum amount of copper which could possibly be extracted. [4]

For economic reasons, mining is abonded once the annual yield falls below 1 000kg. Calculate,

(b) the number of years the mine is in operation, [3]

(c) the total yield of copper during this time. [2]

3. [Paper 2 2011 Q7]

(a) The sum of the first n terms of an arithmetic sequence is given by $S_n = 2n^2 - n$.

(i) Find the first term. [1]

(ii) Find the common difference. [3]

(b) The third term of a geometric progression is 8 and the eight term is $\frac{1}{4}$. Calculate the common ratio. [3]

(c) Given that $\displaystyle\sum_{n=1}^{k}5(3^n) = 442860$, find the value of k. [5]

4. [Paper 2 2009 Q12]

(a) Find the value of $\sum_{r=1}^{20}(-2r+10)$ [4]

(b) Johanna starts a chain letter. She sends four messages concerning the environment to four of her friends. She asks each of her 4 friends to repeat the process by sending the same message to 4 of their friends.

(i) Johanna's 4 letters can be thought of as the first term of a sequence. Consider the information above and calculate the second term of this sequence. [1]

(ii) Calculate how many letters will be written in total, if this chain letter continues and is carried out correctly and without interruption, 6 times where Johanna's 4 letters are considered to be the first term. [2]

(c) Calculate the sum to infinity of the geometric sequence whose first terms are 3^{-1}, 3^{-2} and 3^{-3} Respectively. [2]

5. [Paper 2 2010 Q12]

(a) In an arithmetic progression, the 8th term is twice the 4th term and the 20th term is 40.

(i) Find the common difference *d*. [3]

(ii) Find the sum of all terms from the 8th to the 20th term inclusive. [4]

(b) (i) Evaluate $\sum_{t=1}^{12}(24-3t)$ [3]

(ii) Write down, in terms of *p* where $0 < p < 1$ the first three terms of $\sum_{k=1}^{\infty}27p^k$. [1]

(iii) Find an expression, in terms of p for $\sum_{k=1}^{\infty}27p^k$. [1]

(iv) Find the value of p for which $\sum_{k=1}^{\infty}27p^k = \sum_{t=1}^{12}(24-3t)$ [2]

6. [Paper 2 2012 Q12]

(a) The sum of the first *p* terms of a sequence is given by $p(p + 1)$. Find the 5^{th} term. [5]

(b) Calculate $\sum_{k=4}^{\infty}3^{-k+1}$ [4]

7. [Paper 2 2013 Q12]

(a) The nth term of a sequence T_n is given by $T_n = 4(m-1)^{n+1}$, where *m* is a real number and $m \neq 1$.

(i) Show that the sequence is geometric. [2]

(ii) Determine the values of *m* for which the series $\sum_{n=1}^{\infty}4(m-1)^{n+1}$ converges.

Calculate, in terms of *m*, the sum of infinity of the series. [4]

(b) Find the value of *n* if $\sum_{k=0}^{n}5\times3^k = 1328600$ [4]

(c) The first three terms of an arithmetic progression are 5, *x* and *y* respectively. The first three terms Of a geometric progression are *x*, *y* and 81 respectively. Find the possible values of *x*. [4]

8. [Paper 2 2014 Q12]

(a) The sum of an arithmetic series is 100 times the first term, while the last term is 9 times the first term. Calculate the number of terms in the series if the first term is not equal to zero. [3]

(b) The n^{th} term of a sequence, T_n, is given by $T_n = 3(m-1)^{n+1}$, where m is a constant, $m \neq 1$.

(i) Show that the series is geometric. [2]

(ii) Determine the sum to infinity in terms of m. [2]

9. [Paper 2 2015 Q7]

(a) Find the sum to infinity of the geometric progression whose first term is $\frac{2}{5}$ and whose second term is $\frac{8}{25}$. [3]

(b) An arithmetic sequence is –3, 3, 9…. . The n^{th} term is the first one that exceeds 500. Find the value of n.[3]

(c) Given that $\sum_{k=1}^{n} 2\times 3^{k} = 43046718$, find the value of n. [5]

10. [Paper 2 2016 Q11]

(a) The length of each of the 20 pipes of a church organ is $\frac{15}{16}$ of the length of the adjacent pipe. The length of the longest pipe is 300 cm.

(i) Find the length of the shortest pipe, giving your answer to the nearest cm. [2]

(ii) Find the total length of pipe, in metres, needed to make all 20 pipes. [3]

(b) The sum of the series 11, 14, 17, ….., 68 can also be written as $\sum_{n=x}^{20}(an+b)$.

Find the values of a, b and x. [4]

(c) The first 2 terms of a convergent geometric series are m and 6 respectively. The sum to infinity is 25. Determine the possible non-zero values for m. [5]

Section 15 Inequalities; algebraic fractions; tangents and completion of the square.

1. [Paper 2 2011 Q 11(a)]

Find the set of values of x for which $x^2 - 2x - 2 > 1$ [3]

2. [Paper 2 2010 Q 1]

(a) Solve the inequality $2x^2 + x < 15$ [3]

(b) (i) Express $-2x^2 - 4x + 1$ in the form $A(x + B)^2 + C$, where A, B and C are constants. [3]

(ii) Hence or otherwise, write down the range of $y = -2x^2 - 4x + 1$. [1]

3. [Paper 2 2009 Q7]

The equation of a curve is $y = -x^2 - 4x + 1$.

(a) (i) Express $-x^2 - 4x + 1$ in the form $a(x + b)^2 + c$ whereby a, b and c are constants [3]

(ii) Hence, or otherwise, write down the coordinates of the turning point of the curve. [1]

(iii) Determine whether this turning point is a maximum or a minimum point. [2]

(b) The function f: $x \mapsto -x^2 - 4x + 1$ for $-5 \leq x \leq 0$

(i) Find the range of f. [2]

(ii) State whether the inverse function, f^{-1}, exists. Give a reason for your answer. [2]

4. [Paper 2 2012 Q4]

(a) Express $\dfrac{A}{x+2} - \dfrac{B}{x-3}$ as a single fraction. [3]

(b) Hence, find the values of A and B if $\dfrac{A}{x+2} - \dfrac{B}{x-3} \equiv \dfrac{-2x-9}{(x+2)(x-3)}$ [4]

5. [Paper 2 2008 Q6 (b)]

(i) Solve the equation $y^2 - 4y = 1$, giving your answer correct to 2 decimal places. [3]

(ii) Hence, solve the equation $2^{2x} - 2^{x+2} = 1$. [3]

6. [Paper 2 2013 Q 1]

(a) Solve the inequality $2x^2 + 3x \geq 2$ [3]

(b) **(i)** Express $-2x^2 - 4x + 1$ in the form $a(x + B)^2 + C$, where a, B and C are constants. [3]

(ii) Hence write down the turning point of $y = -2x^2 - 4x + 1$ [1]

7. [Paper 2 2014 Q 5]

The line $y = cx + 4$, where c is a constant, does not intersect the curve $y = x^2 - 2x + 20$.

Find the set of values of c. [5]

8. [Paper 2 2014 Q 7]

The equation of a curve is $y = 2x^2 - 12x + 16$.

(a) **(i)** Express $2x^2 - 12x + 16$ in the form $a(x + b)^2 + \text{c}$, where a, b and c are constants. [3]

(ii) Hence, or otherwise, write down the coordinates of the turning point of the curve. [1]

(iii) Determine whether the turning point is a minimum or maximum point. [2]

(b) The function f is defined by f: $\mapsto 2x^2 - 12x + 16$ for $0 \leq x \leq 5$.

(i) Find the range of f. [2]

(ii) State whether the inverse function f^{-1} exists, giving reason for your answer. [2]

9. [Paper 2 2016 Q1]

(a) Solve the inequality $2x^2 - 13x < -15$ [3]

(b) **(i)** Express $-2x^2 + 12x - 13$ in the form $a(x + p)^2 + q$, where a, p and q are constants. [3]

(ii) Hence write down the turning point of $y = -2x^2 + 12x - 13$. [1]